STUDIES OF THERMAL STRESSES AND STRAIN RATES IN A DISC FOR DIFFERENT MATERIALS

SYNOPSIS

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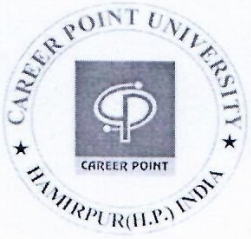

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1. INTRODUCTION

The application of the principles of mechanics to bulk matter is conventionally divided into the mechanics of fluids and the mechanics of solids. The entire subject is often called continuum mechanics. Solid mechanics is concerned with the stressing, deformation and failure of solid materials and structures. Any material, fluid or solid, can support normal forces. These are forces directed perpendicular, or normal, to a material plane across which they act. The force per unit of area of that plane is called the normal stress. Water at the base of a pond, air in an automobile tire, a stretched rubber band and the bones of a runner all support force in that way (some only when the force is compressive). Shearing forces are directed parallel, rather than perpendicular, to the material surface on which they act; the force per unit of area is called shear stress. For example, consider a vertical metal rod that is fixed to a support at its upper end and has a weight attached at its lower end. If we consider a horizontal surface through the material of the rod, it will be evident that the rod supports normal stress. But it also supports shear stress, and that becomes evident when we consider the forces carried across a plane through the rod that is neither horizontal nor vertical. Materials with tightly bound atoms or molecules, like the crystals formed below melting temperature by most substances or simple compounds, or the amorphous structures formed in glass and many polymer substances at sufficiently low temperature, are usually considered solids. But if we picked up and tossed as a ball against a wall, so that large forces act only over the short time of the impact, it bounces back and retains its shape like a highly elastic solid. In the simple but very common case when such a material is loaded at sufficiently low temperature and/or short time scale, and with sufficiently limited stress magnitude, its deformation is fully recovered upon unloading. We then say that the material is elastic. But substances can also deform permanently, so that not all deformation is recovered. For example, if you bend a metal coat hanger substantially and then release the loading, it springs back only partially towards its initial shape, but does not fully recover and remains bent. We say that the metal of the coat hanger has been permanently deformed and in this case, for which the permanent deformation is not so much a consequence of long time loading at sufficiently high temperature, but more a consequence of subjecting the material to large stresses (above the yield stress), we describe the permanent

deformation as plastic deformation, and call the material elastic-plastic. Permanent deformation of a sort that depends mainly on time of exposure to a stress, and that tends to increase significantly with time of exposure, is called viscous or creep deformation and materials which exhibit that, as well as tendencies for elastic response, are called viscoelastic solids (or sometimes visco-plastic solids when we focus more on the permanent strain than on the tendency for partial recovery of strain upon unloading). Solid mechanics, which includes the theories of elasticity, plasticity, and creep and fracture as shown in Figure 1.

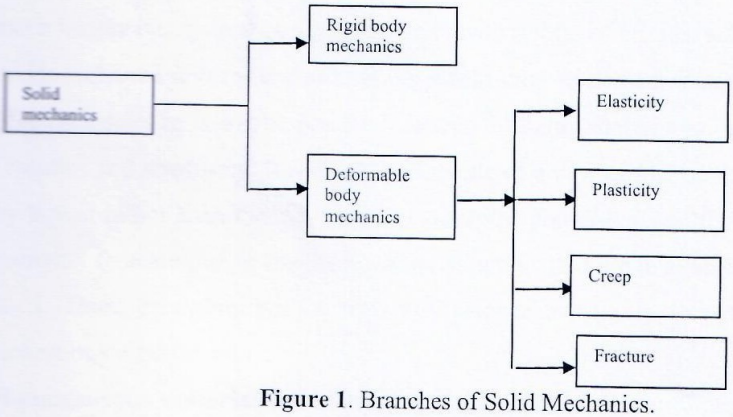

Figure 1. Branches of Solid Mechanics.

Recent developments in the field of solid mechanics, such as in space and nuclear engineering as its ruling parts has required not only the exploitation of superior high temperature and high stress as the hypersonic planes, jet engines, missiles, gas turbines, nuclear reactors etc., but has also required the research of analytical methods of stress and strain for various environments. Considerations of plastic state of matter are interest to many branches of sciences and engineering. The material engineers are interested in plastic flow as it relates forming processes such as rolling, forging, bending, stretching and deep drawing. The design engineers are interested in deformation behaviors for avoiding excessive deflection or distortion in machine parts and engineering structures. The steal and metal workers desires to control more accurately the mechanical process of forming metals at forging temperature. Because of large quantity of energy, consumed in steal mills, at present time during the process of rolling, a more economical use of energy is needed. On the other hand, in order to choose the right materials for various parts of his machines, the machine designers carefully consider the mechanical properties of these materials.

The interdisciplinary field of materials science, also commonly termed materials science and engineering, is the design and discovery of new materials, particularly solids.

The intellectual origins of materials science stem from the enlightenment, when researchers began to use analytical thinking from chemistry, physics and engineering to understand ancient, phenomenological observations in metallurgy and mineralogy. Materials science still incorporates elements of physics, chemistry, and engineering. As such, the field was long considered by academic institutions as a sub-field of these related fields. Beginning in the 1940s, materials science began to be more widely recognized as a specific and distinct field of science and engineering, and major technical universities around the world created dedicated schools for its study. Materials science is a syncretic discipline hybridizing metallurgy, solid-state physics, ceramics and chemistry. It is the first example of a new academic discipline emerging by fusion rather than fission. Many of the most pressing scientific problems humans currently face are due to the limits of the materials that are available and how they are used. Thus, breakthroughs in materials science are likely to affect the future of technology significantly.

Homogeneous materials: A material of uniform composition throughout that cannot be mechanically separated into different materials. Examples of homogeneous materials are certain types of plastics, ceramics, glass, metals, alloys, paper, board and resins.

Isotropic and anisotropy materials: Isotropic material means a material having identical values of a property in all directions. Glass and metals are examples of isotropic materials. Anisotropy materials used in various scientific disciplines to indicate that certain properties of matter vary with the direction from which they are measured. For instance, if the refractive index or density of a material is different when measured along different axes, that property is said to be anisotropic. Anisotropy is the opposite of isotropic, a term used when properties

Orthotropic material: The orthotropic materials, in which there exist two planes of symmetry perpendicular to each other. When a material has two perpendicular planes of symmetry, then the material is called orthotropic e.g. wood (*see* Figure 2) see For steel metal. In wood, we can define three mutually perpendicular directions at each point in which the properties are different. These are the axial direction (along the grain), the radial direction, and the circumferential direction.Because the preferred coordinate system is cylindrical-polar, this type of orthotropy is also called polar orthotropy. Mechanical properties, such as strength and stiffness, measured axially (along the grain) are typically better than those measured in the radial and circumferential directions (across the grain).

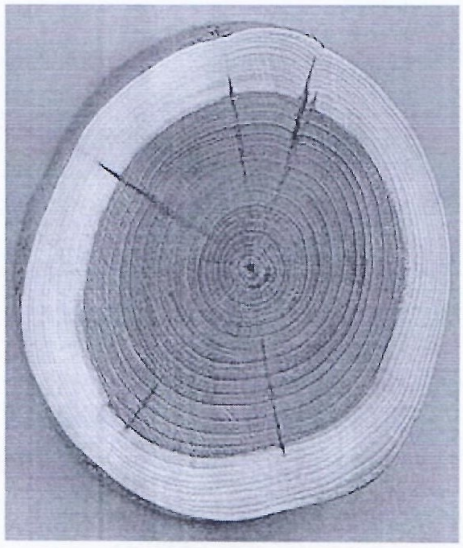

Figure 2. (Wood)

Transversely isotropic material: A transversely isotropic material is one with physical properties that are symmetric about an axis that is normal to a plane of isotropy. This transverse plane has infinite planes of symmetry and thus, within this plane, the material properties are the same in all directions. Hence, such materials are also known as "polar anisotropic" materials. In geophysics, vertically transverse isotropy is also known as radial anisotropy.

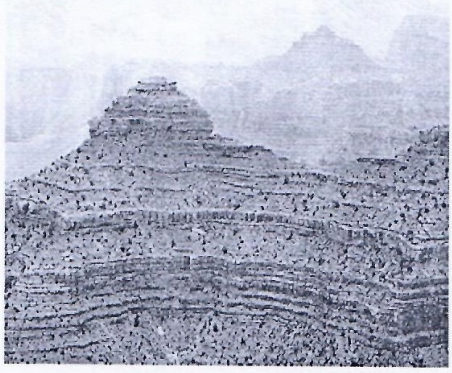

Figure 3. Sedimentary rocks

Transverse isotropy is observed in sedimentary rocks (*see* Figure 3) at long wavelengths. Each layer has approximately the same properties in-plane but different properties through-the-thickness. The plane of each layer is the plane of isotropy and the vertical axis is the axis of symmetry. The symmetry about isotropic, transversely isotropic and orthotropic materials is given Figure 4.

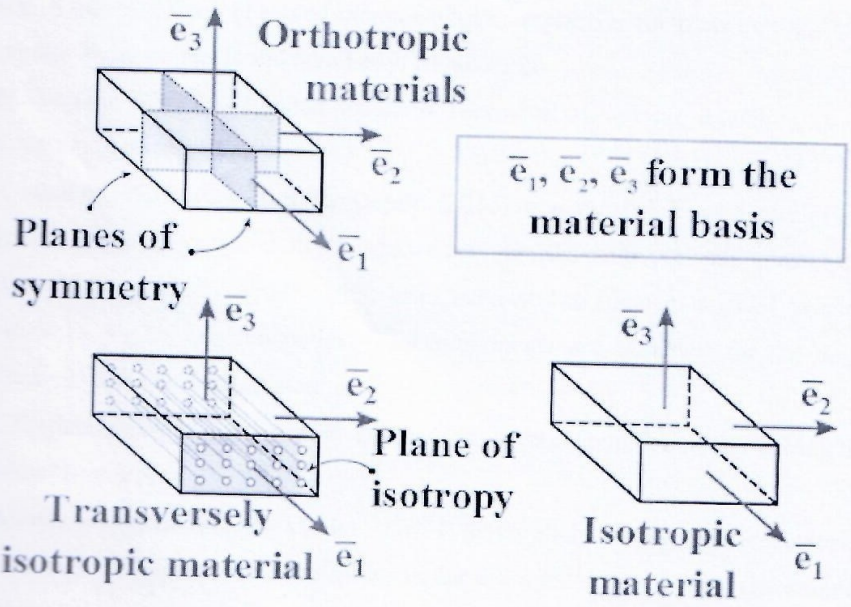

Figure 4. Symmetry isotropic, transversely isotropic and orthotropic materials

2. LITERATURE REVIEW

Lots of authors worked on the mathematical problems of disc under thermal effect condition:

A.G. Temesgen, S.B. Singh and P. Thakur (2020) discussed the problem in Elastoplastic analysis in functionally graded thick-walled rotating transversly isotropic cylinder under a radial temperature gradient and uniform pressure. It is concluded that a functionally graded thick-walled rotating cylinder made of steel subjected to a radial temperature gardient and uniform pressure is on a safer than a cylinder made of titanium, owing to the percentage increase in pressure.

B. R. Seth (1966) investigated the Measure- concept in mechanics. It is shown that the use of generalizes measures helps to bridge the gulf between the microscopic description of physical phenomenon, to eliminate semi-empirical conditions like Tresca and von Mises and creep strain laws, to simplify constitutive equations by prescribing a priori the order of measure of a deformation.

B. N. Borah (1969) investigated the problem of thermo-elastic-plastic transition of shells under uniform pressure and steady state temperature by using Seth transition

theory. It has also been observed that constitutive equations for transition and plastic state grow from the result and take Levy- Mises form.

B. N. Borah (1971) investigated problems Thermo-elastic-plastic transition by using Seth transition theory.

H. Çallıoğlu, M. Sayer and E. Demir (2015) investigated Elastic–plastic stress analysis of rotating functionally graded discs. In this study, elastic–plastic stress analysis of Functionally Graded (FG) discs subjected to constant angular velocity is investigated. Radial and tangential stress components are obtained for the angular velocities and gradient parameters.

K.E. Bisshopp (1944) has analyzed the problem in stress coefficient for rotating disks of conical profile.

N. Ahmet, Eraslan and Y. Orcan (2004) have analyzed parametric analysis of rotating variable thickness elastoplastic annular disks with inner boundaries subject to pressure or mounted on rigid inclusion. Elastic limit angular velocities, partially plastic deformations and plastic limit angular velocities are investigated with emphasis on the determination of the effects of all the geometric, material and hardening parameters involved.

P. Thakur (2009) has analyzed a disc of variable thickness made of compressible material yields at some radius R_1 at a higher pressure as compared to disc made of incompressible material that yields at the outer surface. Flat disc made of incompressible material yields on internal surface at higher pressure as compared to disc made of compressible material. Circumferential stress is maximum at the outer surface of the disc having variable thickness.

P. Thakur (2010) has obtained stresses for the elastic-plastic transition and fully plastic state for a thin rotating disc with rigid shaft at different temperatures. It has been observed that at room temperature rotating disc made of compressible material and of smaller radii ratio yields at the internal surface at a higher angular speed as compared to rotating disc made of incompressible material. With the introduction of thermal effect rotating disc yields at the outer surface at a lesser angular speed as compared to rotating disc at room temperature.

P. Thakur (2010) has calculated creep stresses and strain rates for a thin rotating disc with shaft at different temperature. It has been observed that radial stress has maximum value at the internal surface of the rotating disc made of incompressible

material as compared to circumferential stress and this value of radial stress further increase with the increase of angular speed. Strain rates have maximum values at the internal surface for compressible material.

P. Thakur (2011) have analyzed elastic-plastic transitional stresses in an annular disc having variable thickness and variable Poisson's ratio subjected to internal pressure using Seth's transition theory. It is seen that thickness and passion's ratio variation influence significantly the stresses and pressure required for initial yielding. The thickness variation reduces the magnitude of the stresses and pressure needed for fully plastic state. It is seemed for fully plastic state that circumferential stresses is maximum at the outer surface.

P. Thakur (2012) have obtained stresses for the elastic-plastic transition and fully plastic state for a thin rotating disc with rigid shaft having variable thickness by using Seth's Transition theory . It has been observed that in the absence of thickness, rotating disc with inclusion and made of compressible material e.g. Copper, Brass and Steel, yields at the internal surface at a lesser angular speed as compared to a rotating disc made of incompressible material e.g. rubber whereas it requires a higher percentage increase in angular speed to become fully plastic.

P. Thakur (2013) have obtained stresses for the elastic-plastic transition and fully plastic state has been derived for a thin rotating disc having variable thickness with shaft at different temperatures. It has been observed that in the absence of thickness, the rotating disc made of incompressible material e.g. rubber with inclusion require a higher angular speed to yield at the internal surface as compared to a disc of compressible material e.g. copper, brass. The radial stresses is maximum at the internal surface with no thickness, when thickness varies the circumferential stress is maximum at the internal surface.

P. Thakur, J. Kaur and S. B. Singh (2016) discussed the problem in Thermal creep transition stresses and strain rates in a circular Disc with shaft having variable density. Thermal effect increased value of radial stress at the internal surface of the rotating disc made of incompressible material as compared to tangential stress and this value of radial stress further much increases with the increase in angular speed as compared to without thermal effect.

P. Thakur and M. Sethi (2020) analyzed the problem Creep deformation and stress analysis in a transversely material disc subjected to rigid shaft by using Transition

theory. It has been seen that radial stress has the maximum value at the inner surface of the rotating disk made of isotropic material as compared to the hoop stress and this value of radial stress further increases with the increase in the value of angular speed.

R. K. Shukla (1996) have determined creep stresses in a thin rotating annular disc made of non-homogeneous material by using transition theory. It has been observed that in the presence of non-homogeneity having lesser value at the bore, reduces the stresses and the angular velocity required for steady state of creep as compared to the homogeneous disc.

R.K. Shukla (2000) has obtained elastics-plastic transitional stresses in a non-homogeneous disc with variable thickness subjected to internal pressure by using Seth's transition theory. Non-homogeneity in the disc is due to variation of Poisson's ratio of the material. It is found that the presence of non-homogeneity and thickness variation influence significantly the stresses and pressure required for initial yielding. The thickness variation reduces the magnitude of the stresses and pressure needed for fully-plastic state.

S. Hulsurkar (1979) have analyzed thermo elastic plastic transition in rotating disks with steady state temperature. It has been observed that if the additional condition of incompressibility is imposed, then the expression for stresses correspond to those arising from Tresca yield criterion.

S. K. Gupta and R. K. Shukla (1994) have analyzed the effect of non-homogeneity on elastic-plastic transition in a thin rotating disc. It has been observed that in the presence of non-homogeneity ($k > 0$) in thin rotating discs required higher angular velocity for initial yielding as compare to homogeneous disc.

S. K. Gupta (2000) have analyzed creep stresses and strain rates in a thin rotating disc having variable thickness and variable density by using Seth's transition theory. It has been concluded that a rotating disc whose density and thickness ratio decrease radially is on the safer side of the design in comparison to a flat disc having variable density.

S. B. Singh and S. Ray (2003) have analyzed yield criterion for residual stress and steady state creep in an anisotropic composite rotating disc. Processing of composites containing whiskers and short fibers often result in anisotropy due to preferential alignment of reinforcing element during flow as in extrusion.

S.B. Singh and P. Thakur (2013) has obtained the solution in Elastic-plastic and creep analysis problems in disc. In this research project investigated the problem of

elastic-plastic and creep deformation in disc, cylinder and shell by using transition theory.

S. K. Gupta and P. Thakur (2007) have analyzed rotating disc with inclusion and made of compressible material requires lesser angular speed to yield at the internal surface and higher percentage increase in angular speed to become fully plastic as compare to disc made of in compressible material. Thermal effect also increases the values of radial and circumferential stresses at the internal surface for fully-plastic state.

S. K. Gupta and P. Thakur (2007) have obtained creep stresses and strain rates for a thin rotating disc with inclusion by using Seth's transition theory. It has been observed that radial stress has maximum value at the internal surface of the rotating disc made of incompressible material as compared to circumferential stress and this value of radial stress further increases with the increase in angular speed. Strain rates have maximum values at the internal surface for compressible material.

S. Strashnov, S. Alexandrov and L. Lang (2019) has analyzes description of residual stress and strain fields in FGM hollow disc subject to external pressure The elastic modulus and tensile yield stress of the disc are assumed to vary along the radius whereas the Poisson's ratio is kept constant.

U. Gammer (1994) has obtained the solution of elastic-plastic stress distribution in a rotating annulus and in the annulus under external pressure.

U. Güven (1997) have analyzed fully Plastic rotating disk with rigid inclusion. The plane state of stress in a fully plastic rotating annular disk mounted on a rigid circular shaft is studied. The analysis is based on Tresca's yield condition, its associated flow rule, and linear strain hardening material behaviour.

U. Güven (1998) has obtained elastic-plastic stress distribution of a rotating hyperbolic annular disk mounted on a circular rigid shaft. The exact solution presented is based on the usual assumptions of plane stress, Tresca's yield condition, its associated flow rule and linear strain hardening material behaviour. According to the present analysis the plastic core consists of three parts with different forms of the yield condition. The theory presented is illustrated by numerical results. The results show that the plastic flow and maximum stresses are influenced by the thickness parameter.

U. Güven (1999) has analyzed plane state of stress in an elastic-plastic rotating annular disk mounted on a rigid circular shaft. The analysis is based on Tresca's yield condition, specifically its associated flow rule and linear strain hardening material

behavior. In the present analysis, it is shown that the plastic core adopts three parts, within different forms of the yield condition.

W. Jeong , S. Alexandrov and L. Lang (2018) investigated effect of plastic anisotropy on the distribution of residual stresses and strains in rotating annular disks, The solution for strain rates reduces to one nonlinear ordinary differential equation and two linear ordinary differential equations, even though the boundary value problem involves two independent variables.

3. SIGNIFICANCE OF THE STUDY

This research proposal is of importance in the design of engineering structures e.g. flywheel, turbojet engines, centrifugal compressors, steam and gas turbines, turbo generators, reciprocating engines, brake discs, the casting of ship propellers, etc.

4. PROPOSED METHODOLOGY

In year 1962 Seth investigated transition theory of elastic-plastic deformation, creep and relaxation. This theory does not required any assumptions like an yield condition, incompressibility condition and thus posses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deformed field.

1) **Seth given governing equation by taking following steps**:

- Stress- Strain relations

  $\tau_{ij} = c_{ijkl} e_{kl}$

  where τ_{ij} and e_{kl} are stress and strain tensors; c_{ijkl} are the elastic constants respectively.

- Equations of Equilibrium

  $\frac{d}{dr}(r\tau_{rr}) = \tau_{\theta\theta} - \rho\omega^2 r^2$

  where τ_{rr}, $\tau_{\theta\theta}$ be the radial, circumferential stresses and ρ variable density.

2) Non-Linear differential equation:-

$$n\gamma\beta(p+1)^{n-1}\, dp/d\beta = \frac{\gamma c'}{(1-c)}[1-\beta^n(p+1)^n]\frac{1}{\beta^n} - np[(1-c)+(p+1)^n] + c[1-(p+1)^n]$$

3) Turning points of differential equation are:

p=-1 and p$\rightarrow \pm\infty$

where p is a function of β and β is function of r.

Neglect following assumptions: Incompressibility condition, a power relationship between stress and strain, yield condition such as Tresca or Von-Mises, deformations are assumed to be small to make infinitesimal strain theory applicable.

5. OBJECTIVES OF THE STUDY:

The objective of this research topic is to develop a consistent analytical model capable to resolve a class of control problems for rotating disc for different materials subjected to thermal gradient effect. The novelty of the current research is to be including control factors such as rotating speed, density parameter, thickness, temperature and mechanical load in the consideration of the optimal performance of the disc.

6. SOLUTION THROUGH PROBLEMS

Seth (1962) proposed transition Theory and defined generalized strain measure. Seth's defined transition function for elastic-plastic and creep state. It has been observed that the solid first deforms elastically when a solid is subjected to an external load. If the loading is continued, plastic flow may set in, and if continued further, it give rise to time dependent creep deformation. It may be possible that a number of transition states may occur at the same critical point, then the transition function will have different values. In general, the material from elastic state can go over into (i) plastic state, or to (ii) creep state, or (iii) first to plastic state and then to creep and vice-versa, depending upon the loading. All these final states are reached through a transition state. In the plane stress condition, the transition can take place either through the principal stresses τ_{rr} or $\tau_{\theta\theta}$ becoming critical or through the principal stress difference $\tau_{rr}-\tau_{\theta\theta}$ becoming critical. Hence we have to consider the following three cases:

(*a*) Transition through τ_{rr},

(*b*) Transition through $\tau_{\theta\theta}$,

(*c*) Transition through $\tau_{rr}-\tau_{\theta\theta}$.

For each transition point, we have to determine the stresses and strains corresponding to the above three cases.

7. THESIS OUTLINE:

In the proposed thesis an attempt will be made to study characterization of materials in the disc under various conditions by using transition theory and generalized strain measure. The tentative chapter wise plan of the thesis has been proposed as under:

Chapter 1: Introduction

Chapter 2: Literature review.

Chapter 3: Investigation thermo elastic-plastic transition in rotating disc for isotropic materials.

Chapter 4: Investigation thermo elastic-plastic transition in rotating disc for transversely/orthotropic materials.

Chapter 5: Investigation thermo creep deformation in rotating disc for isotropic materials.

Chapter 6: Investigation thermo creep deformation in rotating disc for transversely/orthotropic materials.

Chapter 7: Conclusions and scope for further research.

8. RESEARCH PLAN SCHEDULE:

Research activity	Progress in months						
No. of Months	0	6	12	18	24	30	36
Course works, extended literature review							
Construction of theoretical framework/model							
Determination of stress and strain/ rates							
Result and discussion							
Publication							
Thesis writing							

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